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TURMOIL THAT WILL NOT ABATE

As I write this comment in early November, the COVID-19 pandemic persists and continues to generate behavioral disruption and political mayhem. Direct, indirect, and unrelated economic and financial events defy reversion to the mean of a more tranquil society. As more than one person has recounted, Isaac Newton was quite productive during the Bubonic Plague years.[1] In this regard, I will emulate the great Newton and, for purposes of this letter, ignore world events to focus on research problems.

TWO CLASSIC PROBLEMS

I've recently come across two classic problems to contemplate. First, I am assisting a friend as he builds a business to implement "modern portfolio theory" (MPT) as intuitively as possible. This MPT is the well known problem of minimizing investment portfolio variance subject to numerous constraints.[2] I'd always considered MPT to be "a solved problem" and, hence, not asking for my attention. Yet real world implementation is challenging! The linear algebra is elegant and the numerical coding is fun. Though my brief recent work does not make me a leading practitioner, it's clear that the MPT calculations share a common bond with so many other world problems. Specifically, the success of an MPT project is limited more by the quality of the input data and information than it is by the mathematics.

My second problem of recent months came from this issue's author Dr. Thomas Little. This *Journal of Derivatives* issue publishes Tom's article "The Free Boundary for the American Put Option." The American put option is a "great problem" because the topic is of immense practical value, the mathematics and numerical analysis are challenging, and it has resisted full and complete solution. There is still no clean, quickly calculable solution for the free exercise boundary. While Dr. Little does not himself find such a full solution, I admire his contributions and will spend a few paragraphs to "add color" as they say on the trading floor.

[1] See, for example, https://www.biography.com/news/isaac-newton-quarantine-plague-discoveries.

[2] See, for example, https://www.investopedia.com/terms/m/modernportfoliotheory.asp.

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FREE BOUNDARY OF THE AMERICAN PUT OPTION

For the many readers who are expert in the history and gymnastics of solution attempts for the American put option, I cite two aspects of Tom Little's article. First, he provides an asymptotic ($t \rightarrow 0$) approximation for the free boundary that I believe to be more accurate than earlier approximations. The derivation path to this result is a discovery of Little's. I base my belief on comparisons of some alternatives with my own numerical calculations.

Second, and more beguiling for me personally, is the Little proposal of an empirical "fit" to the free boundary. I advise the audience to read Little's article for the exposition but will make additional remarks here. First, the fit of the simple two-parameter ansatz is amazingly good over several orders of magnitude in time and for all practical values of risk-free rate r and volatility σ. The two parameters must adjust to serve for different values of r and σ. But one finds the excellent fit over the wide range of time values with properly adjusted parameters.[3]

Little's discovery of this ansatz will have great impact for American put option calculations. Fascinating to me is the question of why it works. The form of the ansatz does not appear to be a solution to the nonlinear, implicit, integral equation that describes the free boundary. This expression is far too complex to permit one to simply plug in the ansatz to see that it solves the equation. Further, we have no theory or conjecture to explain why the ansatz *should* work.

As a mathematical nuance and possibly a clue, Carr, Jarrow, and Myneni proposed in 1992 an approximation for the free boundary that resembles the Little ansatz.[4] A critical difference is that this earlier study wrote the time-dependence as $\sqrt{t}$ within an exponent. Little uses t^{γ} instead such that the exponent of time is one of the two adjustable parameters (γ). Empirically, this exponent parameter remains roughly within the range of 0.40–0.45 (*i.e.*, slightly weaker than an exponent of 0.5). This weaker time dependence is qualitatively reminiscent of the $t \rightarrow 0$ asymptotic behavior of the free boundary in which the dependence is $\sqrt{-t \log t}$.

Thus, there is much room for additional research here. Why does the empirical fit work so well? Is it possible to derive one or both parameters as functions of r and σ? What are the limits to the accuracy of the ansatz?

OUR NEW ARTICLES

The seven articles of this issue begin with "The Free Boundary for the American Put Option" that I've just described. The author, Thomas Little, is the proprietor of Hard Analytics LLC and an adjunct professor of Mathematics at the University of Houston.

Haozhe Su and David Newton of Nottingham Trent University and the University of Bath, respectively, implement their previously developed and deployed "QUAD method" for numerical calculation of option prices. In this study, the authors extend QUAD to underlying processes with no closed-form density or characteristic functions. One concept of the effort is to apply finite difference techniques in a narrow, focused manner. As application and demonstration, this article applies QUAD to the SABR no-arbitrage model.

Abootaleb Shirvani, Yuan Hu, and Svetlozar Rachev (all three of the Texas Tech University) and Frank Fabozzi (EDHEC Business School) apply the mixed Levy subordinated market model to study behavioral finance aspects of dynamic asset pricing theory. The authors introduce a new Levy process by incorporating the form of a mixed geometric Brownian motion. They then derive the probability weighting function that determines "fear and greed" disposition of option traders.

Bing Dong, Jindong Wang, and Wei Xu of Shanghai University of International Business and Economics, Peking University, and Ryerson University, respectively, address computational issues for various risk metrics of variable annuity embedded options. While there is significant analysis in the literature for isolated benefits in variable annuities, this study appears to be the

[3] These are my observations from separate study.

[4] See P. Carr, R. Jarrow, and R. Myneni, "Alternative Characterizations of American Put Options," *Mathematical Finance* 2(2), 87–106, April 1992.

first to calculate risk metrics for multiple benefits in one contract. Hence, this effort is an important contribution to derivative analysis of variable annuities. The authors' numerical technique is the "willow tree" that they have applied successfully to other derivative analyses.

Ping Wu of the Bank of Montreal and Hui Lin of Nanjing University propose a new basket option pricing method that successfully combines previously known techniques of conditioning and moment matching. Numerical evaluation to confirm this method gives promising results. The authors claim that implementation of their method for market practitioners is straightforward.

Vincenzo Russo, Rosella Giacometti, and Frank Fabozzi of Generali Italia S.p.A., University of Bergamo, and EDHEC Business School, respectively, propose closed-form solutions for European options of credit risky bonds. The risk-free rate and credit spread have distinct stochastic models. The study includes the analysis of market price data for options on bonds of borrowers with varying credit strengths and prospective default recoveries.

Qin Emma Wang of Oklahoma State University employs options markets to determine the information content of underlying firms' dividend initiations. Her results, based on abnormal implied volatility spread and skew, imply that informed trading is discernible in the market data—especially in markets with greater liquidity.

Joseph M. Pimbley
Editor

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