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It is well known that the approaches underpinning finance and physics are fairly similar. It is not a coincidence that both Copernicus, who wrote an essential treatise about money, and Newton, who held the post of warden of the Royal Mint for a while, made significant (although underappreciated) contributions to finance. There are at least three reasons for the alignment of physics and finance. First, stochastic processes, such as Brownian motion, are widely used in both fields for modeling dynamics of various stochastic quantities. It is not an accident of history that Brownian motion was formalized and put to good use almost simultaneously—in finance by Bachelier in 1900 and in physics by Einstein in 1905. Thus, from the early 20th century to the present, finance and physics have shared many ideas and methodologies. Second, after the end of the Cold War, many physicists moved to financial engineering, bringing with them unique methods for approaching problems inherent to theoretical and computational physics, and contributing in numerous ways to mathematical and computational finance. Third, the entirely new area of science known as econophysics emerged at roughly the same time. Econophysics combines economics, statistical physics, stochastic analysis, stochastic processes, optimization, numerical methods, and finance; it is an interdisciplinary field that naturally brings physical methods and thinking to economics.

The idea for the special issue (SI) "Physics and Financial Derivatives" came to the guest editors' minds as the result of multiple fruitful discussions with their colleagues with similar backgrounds. All of them started their careers in physics and then, for various reasons, moved to finance. During these discussions, the editors gained a broad view of the undeniable connection between physics and finance, which is very extensive. We thought that for many readers without a physics background, it would be useful to highlight the link between finance and physics and illustrate it by collecting several articles based on approaches borrowed from physics and used to solve significant financial problems. That was the original idea which finally brought to life the SI that you, our reader, are looking at now.

The goal of the SI was to make the link between physics and finance as transparent as possible and to demonstrate the use of various physical methods for solving significant problems related to pricing and risk management of financial derivatives. Articles selected for the SI differ from standard articles on quantitative/mathematical/computational finance in one essential respect. Namely, all of them showcase at least one method or approach previously known in physics, for example, path integrals, or eigenfunction expansions, or random matrices, for example, and explain why this particular method is useful in finance.

We hoped that the SI would help to build a mutually beneficial bridge between modern physics and finance, similar to what is being done in econophysics. Possible areas include financial engineering, quantitative analysis, and computational finance.

We started by asking several journals whether they would be interested in producing such an SI. We were lucky to find *The Journal of Derivatives* (JOD) quickly. Its editor, Dr. Joseph Pimbley, is a former physicist and the author of a well-known paper ("Physicists in Finance," *Physics Today*, Jan. 1997). He was excited to collaborate with us and provided his support and assistance. Accordingly, the SI solicited high-quality original research articles and reviews in the area of physical methods applied to financial derivatives. The areas of interest were aligned with, but not restricted to, the topics frequently appearing in JOD.

Finally, after a thorough peer-review process, six articles were accepted. They are briefly presented next in alphabetical order.

In "Physics and Derivatives: *Effective-Potential Path-Integral Approximations of Arrow-Debreu Densities*," L. Capriotti and R. Vaia show how effective-potential path-integral methods, successfully employed in physics for a variety of quantum thermodynamics applications, can be used to develop an accurate and easy-to-compute semi-analytical approximation of transition probabilities and Arrow-Debreu densities for arbitrary diffusions. The authors illustrate the accuracy of the method by presenting results for the Black-Karasinski and the GARCH linear models, for which the proposed approximation provides remarkably accurate results for multiyear time horizons, even in regimes of high volatility. The accuracy and the computational efficiency of the method make it a viable alternative to fully numerical schemes for a variety of derivatives pricing applications.

In "Semi-Closed Form Prices of Barrier Options in the Time-Dependent CEV and CIR Models," P. Carr, A. Itkin, and D. Muravey derive closed-form prices for barrier options written on the underlying driven by a one-factor stochastic model with time-dependent coefficients. The article, which covers both the CIR model for zero-coupon bonds and the constant elasticity of variance model for stocks, implements two approaches. One approach is the generalization of the method of heat potentials for the Bessel process. The second approach is the generalized integral transform, extended to the Bessel process. The authors show that these approaches are more efficient than the backward and forward finite difference methods, while providing better accuracy and stability. It turns out that the problems under consideration can also be formulated in terms of stationary and nonstationary heat transfer. They are similar to problems arising in diffusion, sedimentation, viscous flows accompanied by various kinetic processes, astronomy, atomic physics, absorption, combustion, phase transitions, and many others.

In "Model-Free Backward and Forward Nonlinear PDEs for Implied Volatility," P. Carr, A. Itkin, and S. Stoikov discuss the efficient computation of the Black-Scholes implied volatility. The authors derive backward and forward quasilinear parabolic partial differential equations (PDEs) that govern the implied volatility of a contingent claim whenever the latter is well defined. Additionally, they obtain a forward nonlinear hyperbolic PDE of the first order, which also governs the evolution of the implied volatility. They develop an iterative numerical method for solving the PDEs via finite differences and compute the implied volatility. The approach is based on splitting the PDE operator—an idea first proposed for solving some physics problems. The authors provide a short historical overview to reveal and underline this connection. The computational results obtained in the article support the original intuition that the performance of the finite difference solver exceeds that of the traditional approach.

In "Quantum Option Pricing and Quantum Finance," S. Focardi, F. J. Fabozzi, and D. Mazza discuss quantum probability (the probability theory of quantum mechanics for option pricing and for finance in general) and highlight their motivations to apply quantum probability to finance. The critical issues are quantization, self-reflexivity of markets, and the existence of incompatible observations. The authors outline quantum probability theory, quantum stochastic processes, and the pricing of options in a quantum context. Key findings of the article are (1) quantum probability theory is a probabilistic theory of observations (observations can change the system and be incompatible), (2) quantum probability offers more empirically faithful handling of large-scale events and uncertainty, and (3) quantum probability theory is a better theory of valuation than the classical probability theory.

In "QLBS: *Q-Learner in the Black-Scholes(-Merton) Worlds*," I. Halperin presents a discrete-time option pricing model that is rooted in reinforcement learning (RL), and more specifically in the popular Q-Learning method of RL. The author constructs a risk-adjusted Markov Decision Process for a discrete-time version of the classical Black-Scholes-Merton model, where both the price and hedge are parts of the same formula. Pricing is done by learning to dynamically optimize risk-adjusted returns for an option replicating portfolio, as in the Markowitz portfolio theory. Using Q-Learning and related methods, once created in a parametric setting, the model is able to go model-free and learn to price and hedge an option directly from data generated from a dynamic replicating portfolio, which is rebalanced at discrete times, and without an explicit model of the world. Further, the author suggests using the proposed model for benchmarking different RL algorithms for financial trading applications. Also, the author shows that the option pricing boils down to the optimization problem that is similar to variational methods of Hamiltonian mechanics and stochastic Hamilton-Jacobi-Bellman optimal control in continuous time.

In "Physics and Derivatives: *On Three Important Problems in Mathematical Finance*," A. Lipton and V. Kaushansky use a recently developed extension of the classical heat potential method to solve three crucial, but seemingly unrelated problems of financial engineering: (a) American put pricing, (b) default boundary determination for the structural default problem, and (c) evaluation of the hitting time probability distribution for the general time-dependent Ornstein-Uhlenbeck process. The authors show that all three problems reduce to analyzing the behavior of a standard Wiener process in a semi-infinite domain with a quasi-square-root boundary. The authors rely on the method of heat potentials, which is a robust approach in mathematical physics, used for decades in several relevant fields including heat transfer, the Stefan problem, nuclear engineering, and materials science.

The last article in the SI is not a research article as such. Instead, it is a collection of interviews with several former physicists who have made successful transitions from physics and applied mathematics to finance. While the interviewees joined the field at different times—Derman, Gershon, and Lipton early on, and Antonov, Lorig, Tankov, and Guerrero more recently—their experiences are equally instructive. The editors asked all interviewees the same 12 questions, which, at least to us, seemed to be necessary for gaining a better understanding of the connection between modern physics and finance, at both the professional and human levels. Although encouraged to respond to all the questions, the interviewees were not required to do so. We hope that the readers will benefit from learning about their career trajectories and accomplishments.

We thank all the authors for their contributions and the referees for their diligent and timely reviews. We are also grateful to Dr. Joseph Pimbley, Editor of *The Journal of Derivatives*, for his profound attention, close involvement, and encouragement. Finally, we wish everyone a fruitful and enjoyable reading!

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